Algebra II
Mid Term Examination

Instructions: All questions carry equal (non-zero) marks. Justify all your answers.

1. Let A, B, D be square matrices of size n and let 0 denote the zero matrix. Prove that

$$\det \begin{bmatrix} A & B \\ 0 & D \end{bmatrix} = \det(A)\det(D)$$

2. Let A be an $n \times n$ matrix with integer entries a_{ij}. Prove that A^{-1} has integer entries if and only if $\det(A) = \pm 1$.

3. Let i denote an element whose square is -1. Prove that the set $\{a + bi \mid a, b \in \mathbb{Z}/3\mathbb{Z}\}$ is a field under natural addition and multiplication.

4. Let V be a finite demensional vector space over a field F. Prove that any linearly independent subset of V is a subset of a spanning linearly independent subset of V.

5. Show that the subset $W = \{(x_1, \ldots, x_n) \mid x_1 + 2x_2 + \cdots + nx_n = 0\}$ of $\mathbb{R}^n$ is a subspace and find a basis for W.

6. Prove that a square matrix with entries from a field F is invertible if and only if its coulmns are linearly independent.